



Fig. 1. Elastoplastic Equation of State

where  $K$  is bulk modulus,  $\mu$  is rigidly modulus,  $\nu$  is Poisson's ratio,  $d\epsilon_x = d\rho/\rho$  is incremental strain in the  $x$ -direction and  $\rho$  is density. For  $\nu = \text{const.}$ , which is assumed throughout the following, Eq. (5) can be integrated directly:

$$p_x = 3\bar{p}(1 - \nu)/(1 + \nu). \quad (6)$$

Since strain is uniaxial,

$$dp_y = (K - 2\mu/3)d\epsilon_x, \quad (7)$$

and the incremental change in  $\tau$  is

$$d\tau = (dp_x - dp_y)/2 = 3d\bar{p}(1 - 2\nu)/2(1 + \nu) \quad (8)$$

or

$$\tau = 3\bar{p}(1 - 2\nu)/2(1 + \nu). \quad (9)$$

At  $a$ :

This is the initial yield point, so

$$Y_a = 2\tau_a = 3\bar{p}_a(1 - 2\nu)/(1 + \nu). \quad (10)$$

Along  $ae$ :

Equation (4) is integrated to obtain

$$p_x - p_{xa} = \bar{p} - \bar{p}_a + \frac{2}{3}(Y - Y_a). \quad (11)$$

At  $e$ :

As unloading begins, yield ceases, and

$$Y = Y_e. \quad (12)$$

Along  $ef$ :

This is the unloading phase;  $\tau$  diminishes and changes sign, and the material once again behaves elastically. Equation (5) integrates to

$$p_{xe} - p_x = 3(1 - \nu)(\bar{p}_e - \bar{p})/(1 + \nu).$$

The integral of Eq. (8) is

$$Y_e - 2\tau = 3(\bar{p}_e - \bar{p})(1 - 2\nu)/(1 + \nu).$$

Combining this with Eq. (12) yields

$$Y_e - 2\tau = (p_{xe} - p_x)(1 - 2\nu)/(1 - \nu) \quad (13)$$

At *f*:

This is the unloading yield point;

$$p_y - p_x = Y_f = -2\tau_f \quad (14)$$

and from Eq. (13),

$$Y_e + Y_f = (p_{xe} - p_{xf})(1 - 2\nu)/(1 - \nu) \quad (15)$$

$$p_{xe} - p_{xf} = \frac{1 - \nu}{1 - 2\nu} (Y_e + Y_f).$$

In the special case  $Y_f = Y_e$ ,  $\nu = 1/3$ ,

$$p_{xe} - p_{xf} = 4Y_e.$$

Along *fb*:

Here we have the integral of Eq. (4):

$$p_x - p_{xf} = \bar{p} - \bar{p}_f - \frac{2}{3}(Y - Y_f). \quad (16)$$

At *b*:

The resolved shear stress, calculated elastically, is equal to half the yield stress:

$$Y_b = 2\tau_b = 3\bar{p}_b(1 - 2\nu)/(1 + \nu). \quad (17)$$

Equations (5) through (17) provide means for calculating the stress-strain cycle of Fig. 1 if  $\bar{p}(\rho)$ ,  $Y(\bar{p})$ , and  $\nu$  are known. The extent to which  $\nu$  may vary during such a cycle is presently unknown.

The value of the yield strength is assumed to vary with the pressure according to the relation

$$Y = Y_0 + M(\bar{p} - \bar{p}_a). \quad (18)$$

The mean pressure is assumed to be related to the density by the expression

$$\bar{p} = A\eta + B\eta^2 + C\eta^3 \quad (19)$$

where  $\eta = (\rho/\rho_0) - 1$  and  $\rho_0$  is density at  $\bar{p} = 0$ . The sound speed is

$$c = \sqrt{dp_x/d\rho} = [-3V^2(1 - \nu)(d\bar{p}/dV)/(1 + \nu)]^{1/2} \quad (20)$$

Sound speed is assumed constant,  $c = c_0$ , along *ba* of Fig. 1. Integrating Eq. (20) under this assumption yields

$$\bar{p} = [c_0^2(1 + \nu)/3(1 - \nu)](V^{-1} - V_0^{-1}). \quad (21)$$

Then, at *a*,

$$\eta_a = 3\bar{p}_a V_0(1 - \nu)/c_0^2(1 + \nu). \quad (22)$$

Sound speed on the segments *ae*, *fb* of Fig. 1 is defined by the slopes of the curves shown:

$$c_p^2 = dp_x/d\rho = d\bar{p}/d\rho \pm \frac{2}{3} dY/d\rho. \quad (23)$$

The values of the coefficients in Eq. (19) are obtained by assuming that Hugoniot equation of state data lie on the upper curve of Fig. 1. Values of  $Y_0$  and  $M$  must then be assumed, see Eq. (18). The value of  $Y_0$  corresponds to that obtained statically, and the value of  $M$  is esti-